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DIFFRACTION OF A PLANE WAVE BY A WEDGE MOVING WITH SUPERSONIC SPEED

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We study the perturbation of the uniform stream behind an oblique shock wave that simultaneously diffracts with an incident wave. The deformation of the shock causes the assignment on its shape of a relation in partial derivatives of the unknown pressure perturbation, which determines the formulation of a Hilbert boundary-value problem for an analytic function.

The classical "problem of diffraction of a plane wave" (by a stationary wedge of finite opening angle), which was solved in 1933 [1], is complicated by assuming that the wedge moves through the gas at supersonic speed.

The problem was briefly considered earlier by the author [2]; an integral of Cauchy type was used to construct its solution. It proves to be convenient here to use the generalization obtained by the author [3] of the solution of a diffraction problem that was constructed in [4, 5]; on it are based the considerations and calculations of the pressure distribution of the wedge surface that are contained in the present paper.

For the special case of a thin wedge moving at hypersonic speed, when Lighthill's solution can be used, the examination was carried out in [6, 7]. Conditions under which interaction is realized without diffraction were indicated in [8]; the analysis performed in [9] was devoted to their small perturbations.

1. Flow field. A wedge of finite opening angle β moves with supersonic speed $W_\infty = M_\infty a_\infty$ in a quiescent ideal gas, forming an attached oblique shock wave that forms an angle α with its symmetry plane. At the instant $t = 0$ it meets the front of a weak plane pressure jump that is propagating through the same gas with a speed a_∞ equal to the speed of sound and making an angle φ with the oblique shock front. The resulting motion is self-similar. The magnitude ε of the pressure jump in the incident wave, referred to the pressure in the quiescent gas, is chosen as the basic small parameter.

Considerations are carried out in the plane perpendicular to the edge of the wedge, where the shock plane and symmetry plane of the wedge are represented by a shock line and symmetry line.

A typical picture of the flow arising for $t > 0$ is shown in Fig. 1. The gas particle lying at the tip of the wedge at $t = 0$ is displaced along the surface a distance $Ma_1 t$, if a_1 is the speed of sound and Ma_1 the supersonic speed of the stream relative to the wedge in the region between the wedge and the oblique shock wave (region 1 in Fig. 1). This particle is the center of the Mach circle, which is the leading edge of the resulting perturbation.

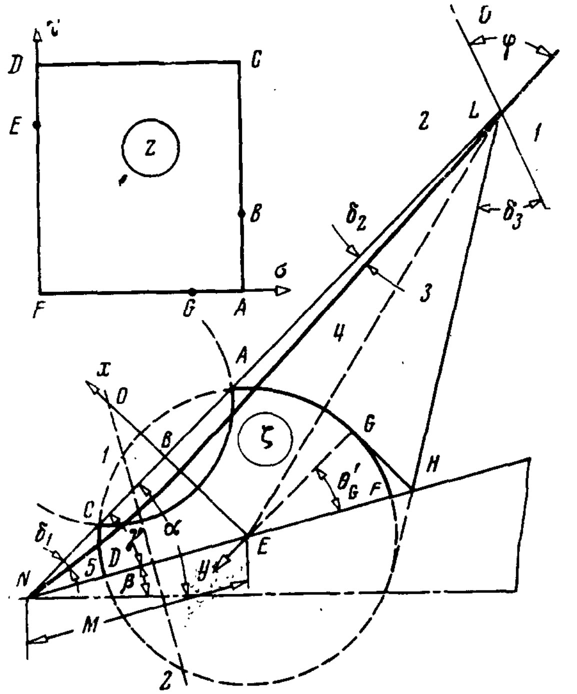

Fig. 1

The flows of the two sides of the symmetry line are independent. The oblique shock wave is a straight shock relative to the gas in regions 0 and 1 (Fig. 1), propagating through them with super- and subsonic speeds respectively. Therefore the resulting perturbations cannot penetrate into region 0, but they necessarily reach the shock on the side of region 1 and, superimposing on one another, cause its weak distortion in the section between the points of intersection of the shock line with the Mach circle.

However to formulate the boundary-value problem on the basis of the smallness of the perturbations, the boundary of the diffraction region is taken as the section ABC of the undisturbed shock, and also the section DEF cut off of the wall by the Mach circle, together with its arcs AF and CD.

The incident wave and the oblique shock, interacting, are refracted at the point of intersection L of their fronts by the finite angle δ_3 and the small angle δ_2, respectively. An examination of such an interaction of shock waves is contained in the work of the author [10]; here it is necessary only to mention that regions 3 and 4 between the fronts of the refracting waves are divided by the line LE of tangential discontinuity.

The front of the refracted wave either touches the arc FA at some point G, or is reflected from the wall so that this arc is touched at the point G by the reflected front, as shown in Fig. 1. Finally, it is possible that the wave that reflects from the wall interacts for a second time with the oblique shock wave, for a second time the refracted wave is reflected anew from the wall, and so on. In all cases, along the arc FA are adjoined to the diffraction region $ABCDEFA$ two regions of uniform flow divided by a weak wave.

The cases also yield to examination in which the normal velocity of the front of the incident wave has a component in the direction of motion of the wedge, but diffraction will take place only under conditions such that the point of intersection of the continuation of the undisturbed incident front with the symmetry line moves faster than the wedge $\csc \chi > M_\infty$, where $\chi = \varphi + \alpha$ is the angle of inclination of the incident front to the symmetry line.

From geometric and kinematic considerations it is not difficult to determine the angle δ_3 of refraction of the incident wave at point L

$$\delta_3 = \varphi - \arcsin [h^{-2}(h^2 - m^2 + \sqrt{h^2 - 1})]$$

Here

$$h = \frac{\sqrt{h_1^2 (1 + M_\infty \sin \chi)^2 + M^2 \sin^2 \varphi - 2h_1 M \sin \varphi \, (1 + M_\infty \sin \chi) \cos \gamma}}{\sin \varphi}$$

$$h_1 = a_\infty / a_1 = (\varkappa + 1) M_c / \sqrt{[2\varkappa M_c^2 - (\varkappa - 1)] \, [2 + (\varkappa - 1) M_c^2]}$$

$$M_c = M_\infty \sin \alpha, \qquad m = M \sin \gamma, \qquad \gamma = \alpha - \beta$$

The position of point G is fixed by the angle $\theta_G' = \pi / 2 - \gamma - \varphi + \delta_3$. The incident wave produces in region 2 behind its front a motion of the gas with speed w, and changes the speed of sound and density there. The values of these quantities are determined by the equations [11]

$$w = \frac{\varepsilon p_\infty}{a_\infty \rho_\infty}, \qquad a_2 = a_\infty + \frac{\varepsilon (\varkappa - 1) p_\infty}{2 a_\infty \rho_\infty}, \qquad \rho_2 = \rho_\infty + \frac{\varepsilon p_\infty}{a_\infty^2} \tag{1.1}$$

Here $\varkappa$ is the polytropic exponent, and the subscript ∞ corresponds to region 0.

With respect to the gas in region 0 the oblique shock wave is a plane shock wave propagating with speed U and inducing behind its front motion of gas of density ρ_1 and pressure p_1 with speed V; the quantities p_1, ρ_1 and V are [4, 11]:

$$p_1 = \frac{2\rho_\infty}{\varkappa + 1} \left(U^2 - \frac{\varkappa - 1}{2\varkappa} a_\infty^2 \right), \qquad \rho_1 = \frac{\varkappa + 1}{\varkappa - 1} \rho_\infty \left(1 + \frac{2}{\varkappa - 1} \frac{a_\infty^2}{U^2} \right)^{-1}$$

$$V = \frac{2}{\varkappa + 1} U \left(1 - \frac{a_\infty^2}{U^2} \right), \qquad U = M_c a_\infty = M_\infty a_\infty \sin \alpha \tag{1.2}$$

Reverting to the real flow, it is necessary to bear in mind that in spite of the assumption of small intensity of the incident wave the scheme described may not always be adequate, since for large angles φ the interaction can lose its regular character; the same is true of reflection of the wave from the wall.

For near critical values, reflection and interaction of the waves induces nonlinear effects [12]. However in a wide range of determining parameters the critical angles between the incident front and the wall will, because of its refraction by interaction, be greater than for diffraction from a stationary wedge. The large number of determining parameters does not permit indicating this range in advance; for each specific case orientation in this question is achieved with the aid of the work [10], which presents the limits of regular reflection and interaction.

2. Formulation of boundary-value problem. In the plane perpendicular to the edge of the wedge it is convenient to associate a system of dimensionless rectangular coordinates with the gas particles in region 1 (cf. Fig. 1), locating its origin at point E and directing the x-axis perpendicular and the y-axis parallel to the line

of the undisturbed oblique shock. The coordinates x, y may be regarded as obtained from the physical coordinates x', y', t according to the equations
$x = x' / a_1 t$, $y = y' / a_1 t$.

For the dimensionless perturbation pressure, density, and components of gas velocity along the x- and y-axes,

$$p = p' / \rho_1 a_1^2, \quad \rho = \rho' / \rho_1, \quad u = u' / a_1, \quad v = v' / a_1$$

where p', ρ', u', v' are the dimensional perturbations, the equations of plane unsteady self-similar gas motion have, after elimination of the function ρ, the form [4, 13]

$$x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\partial p}{\partial x}, \quad x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = \frac{\partial p}{\partial y} \quad (2.1)$$

Conditions on the weakly distorted shock with equation of the front $x = m + f(y)$, where $f(y)$ is of order ε, give together with (1.1) the following expressions for the perturbations:

$$u = \frac{2}{\varkappa+1}\left[(1+M_c^{-2})(f - yf') + \frac{\varepsilon a_\infty}{\varkappa a_1}\left(\frac{\cos\varphi}{M_c^2} - \frac{\varkappa-1}{2}\cos\varphi - \frac{\varkappa-1}{M_c}\right)\right]$$

$$v = -M_1 f' + \frac{\varepsilon a_\infty}{\varkappa a_1}\sin\varphi \qquad \left(M_1 = \frac{V}{a_1} = \frac{2M_c}{\varkappa+1}\frac{a_\infty}{a_1}(1 - M_c^{-2})\right) \quad (2.2)$$

$$p = \frac{4}{\varkappa+1}\frac{a_\infty\rho_\infty}{a_1\rho_1}\left[(M_c(f - yf') + \frac{\varepsilon a_\infty}{\varkappa a_1}\left(M_c\cos\varphi + \frac{M_c^2}{2} - \frac{\varkappa-1}{4}\right)\right]$$

On the surface of the wedge, a condition on the normal derivative $\partial p / \partial n = 0$ is obtained from the third equation of the system (2.1). On the section GF of the arc FA (Fig. 1) the pressure perturbation is either double compared with that in region 3, or is absent if the front of the refracted wave touches the Mach circle without reflection from the wall. From (1.1) it follows that

$$u_3 = \cos(\varphi - \delta_3)p_3, \quad v_3 = \sin(\varphi - \delta_3)p_3 \quad (2.3)$$

where the subscripts $3, 4$, and (later) 5 refer to perturbations. The property of the line of tangential discontinuity leads to the condition

$$(u_3 - u_4)\sqrt{h^2 - m^2} = (v_4 - v_3)m \quad (2.4)$$

After substitution of $f(y) = -(y + \sqrt{h^2 - m^2})\delta_2$, which corresponds to the section AL of the oblique shock, the relations (2.2) together with (2.3) and (2.4) form a system of linear equations for the quantities δ_2, u_3, u_4, v_3, v_4 and $p_3 = p_4$. The equations giving their solution are omitted because of their awkwardness. The same relations (2.2), after substitution of $f(y) = (y - M\cos\gamma)\delta_1$, corresponding to the section NC of the oblique shock, determine the unknown perturbations along the arc CD (in region 5). The quantity δ_1 is determined by the relation

$$\delta_1 = M_\infty \sin 2\alpha\left[\operatorname{tg}\alpha - \operatorname{tg}\beta\left(\frac{\varkappa-1}{2}\operatorname{tg}^2\alpha + \frac{\varkappa+1}{2}\right)\right]HM' -$$

$$-\frac{1}{2}\frac{\sin 2\alpha}{\cos^2\beta}\left[\operatorname{tg}^2\alpha\left(1 + \frac{\varkappa-1}{2}M_\infty^2\right) + 1 + \frac{\varkappa+1}{2}M_\infty^2\right]H\beta'$$

$$H^{-} = \operatorname{tg}\beta\left[3\operatorname{tg}^2\alpha\left(1 + \frac{\varkappa-1}{2}M_\infty^2\right) + 1 + \frac{\varkappa+1}{2}M_\infty^2\right] - 2\operatorname{tg}\alpha(M_\infty^2 - 1)$$

obtained from the perturbations from the equation expressing β in terms of M_∞ and α

[11]. The quantities M' and β' are easily found by means of Eq. (1.2)

$$M' = \frac{\varepsilon}{\varkappa}\left(\sin\chi - \frac{\varkappa-1}{2}M_\infty\right), \qquad \beta' = -\frac{\varepsilon\cos\chi}{\varkappa M_\infty}$$

Note 2.1. The values of the quantities $p_3 = p_4$ and δ_2 can be determined exactly by means of the paper [10], and the quantities δ_1 and p_5 by means of Eq. (1.2) and the equations of the work [14].

3. Reduction to Hilbert problem. It is known [1, 4, 13] that the system of equations (2.1) is, after elimination of the functions u and v, introduction of the polar coordinates r and θ ($x = r\cos\theta$, $y = r\sin\theta$), and transformation of the radius vector $r = 2R/(1+R^2)$, transformed to the Laplace equation for the function p inside the unit circle. Smyrl showed [13] that a contact discontinuity does not invalidate this transformation, because the second derivative of p normal to it remains discontinuous.

The boundary of the diffraction region is thus deformed only on the section AC of the straight line $r = m\sec\theta$, which transforms into the circle $2R\cos\theta = m(1+R^2)$, intersecting the circle $R = 1$ orthogonally in the points A and C. The boundary conditions on this circle lead to condition on the single function p, obtained from the system (2.2) with the aid of the first two equations (2.1) [3, 4, 5]

$$(\partial p/\partial n)/(\partial p/\partial s) = (Am\,\mathrm{tg}\,\theta - B\,\mathrm{ctg}\,\theta)/\sqrt{1 - m^2\,\mathrm{sc}^2\,\theta} \tag{3.1}$$

$$A = \frac{M_c^2+1}{2M_c^2}\left(\frac{2\varkappa M_c^2 - (\varkappa-1)}{2+(\varkappa-1)M_c^2}\right)^{1/2} \qquad B = \frac{\varkappa+1}{2}\,\frac{M_c^2-1}{2+(\varkappa-1)M_c^2}$$

Here n and s are coordinates along the outer normal and tangent to the contour of the diffraction region in the vatiables R, θ.

From consideration of Fig. 1 it is evident that a linear-fractional transformation, which moves points 1 and 2 to the origin and infinity, respectively, transforms conformally the diffraction region in the plane $\zeta = R\exp i\theta$ onto the concentric semi-ring, since the circle $2R\cos\theta = m(1+R^2)$ and the diameter DEF are orthogonal to the straight line $1-2$ and the unit circle, which cross on rays coming from the origin. Multiplication by a constant and subsequent taking of the logarithm of this transformation in superposition determine the function

$$z = \ln\frac{\zeta - \exp i\theta_2}{\zeta - \exp i\theta_1} - i\,\frac{\theta_2-\theta_1}{2}, \qquad \begin{array}{l}\theta_1 = \arcsin M^{-1} - \gamma \\ \theta_2 = \pi - \arcsin M^{-1} - \gamma\end{array} \tag{3.2}$$

and the rectangular region of diffraction in the plane $z = \sigma + i\tau$

$$\begin{array}{l}0 < \sigma < l, \\ 0 < \tau < \pi,\end{array} \quad l = -\frac{1}{2}\ln q, \quad q = \frac{1 - \mathrm{tg}\,\gamma\sqrt{M^2-1}}{1+\mathrm{tg}\,\gamma\sqrt{M^2-1}} \tag{3.3}$$

The form of the reflected front is shown by the right vertical side $\sigma = l$ and the form of the wall by the left side ($\sigma = 0$). The relation $\theta_G = \theta_G' - \frac{1}{2}\pi - \gamma$ and Eqs. (3.2) determine the point G in the z-plane

$$\sigma_G = \tfrac{1}{2}\ln\{[1-\cos(\theta-\theta_2)]/[1-\cos(\theta-\theta_1)]\}, \quad \tau_G = 0 \tag{3.4}$$

Substitution of the inverse of the transformation (3.2) at ($\sigma = l$)

$$\mathrm{tg}\,\theta = m_0 M^{-1}\,\mathrm{ctg}\,\gamma\,(m_0 - M\cos\tau)/(M - m_0\cos\tau), \qquad m_0 = \sqrt{1-(M^2-1)\,\mathrm{tg}^2\gamma}$$

into the right side of (3.1) gives the condition on AC in the z-plane

$$\frac{\partial p / \partial s}{\partial p / \partial n} = b(\tau) = \frac{P}{Q} = \frac{\operatorname{tg}\gamma m_0^2 M \sqrt{M^2-1}\,(m_0 - M\cos\tau)\sin\tau}{m m_0^2 A\,(m_0 - M\cos\tau)^2 - BM^2 \operatorname{tg}^2\gamma\,(M - m_0\cos\tau)^2} \quad (3.5)$$

In the z-plane $\partial p/\partial s = \partial p/\partial\sigma$ on CD and FA, and $\partial p/\partial s = \partial p/\partial\tau$ and $\partial p/\partial n = \partial p/\partial\sigma$ on AC and DF. Therefore the conditions on $CDFA$, the assignment of piecewise constant values of p on CD and FA and zero value of $\partial p / \partial n$ on DEF, can be written by means of the delta function as the single relation $\partial p / \partial\sigma = p_4\delta(z - z_G)$, whose right side expresses the doubling of the pressure on passing through the point G along the arc AF.

The function p must satisfy two more conditions. The first of them is obtained by integrating with respect to y along the image of the shock front the relation obtained from (2.2), and the meaning of the second is clear at once from its writing; in the variables σ and τ this condition has the form

$$B\int_0^\pi \frac{\partial p}{\partial\tau}\frac{d\tau}{y(\tau)} = v_5 - v_4, \qquad \int_0^\pi \frac{\partial p}{\partial\tau}\,d\tau = p_5 - p_4 \quad (3.6)$$

Introducing the coefficients P, Q and S and letting $P / Q = b(\tau)$ and $S = 0$ on AC and $P = 1$, $Q = 0$ and $S = p_4\delta(z - z_G)$ on $CDFA$, it is possible finally to represent the whole system of boundary conditions by the single relation $P\partial p / \partial\sigma - Q\partial p / \partial\tau = S$, which sums up the formulation of the nonhomogeneous Hilbert boundary-value problem for the function $\Gamma(z) = \partial p / \partial\sigma - i\partial p / \partial\tau$, analytic in the rectangle (3.3).

4. Homogeneous problem. Method of Lighthill. The solution of the homogeneous problem, obtained from that formulated in Sect. 3 by supposing that $S = 0$ everywhere on the contour, is denoted below by $\Gamma_0(z) = \partial p^\circ / \partial\sigma - i\partial p^\circ / \partial\tau$ and is assumed continuous in the region (3.3) up to and including the boundary.

It can be verified that from (3.5) is obtained the representation for the argument of $\Gamma_0(z)$ on the image AC of the distorted section of the reflected front

$$\arg\Gamma_0(z) = \sum_{j=1}^{4} \operatorname{arc\,tg}\left(E_j \operatorname{tg}\frac{\tau}{2}\right) \quad (4.1)$$

$$E_{1,2,3,4} = \sqrt{(M+m_0)/(M-m_0)}\,(D_{1,2} \pm \sqrt{D_{1,2}^2 - 1})$$

$$D_{1,2} = \frac{1 - m^2 \pm \sqrt{1 - m^2 - 4mB\,[(1-m^2)A - mB]}}{2\,[(1-m^2)A - mB]}$$

The function $\Gamma_0(z)$ is found as the product of a constant subject to determination and of the functions analytic in (3.3) $\Gamma_0(z) = c\Lambda(z)L(z)$, where $\arg\Lambda(z) - 2\pi$ is equal to the right side of (4.1) on AC and zero on $CDFA$, and $\arg L(z)$ is equal to 2π on AC and $3\pi / 2$ and $\pi / 2$ respectively on the parts of the section $CDFA$ from point C to the point $z = z_0$ and from point $z = z_0$ to point A.

It is not difficult to show that the Fourier sine series for the function $\arg\Gamma_0(l + i\pi) - 2\pi$ in the interval $0 < \tau < \pi$ obtained from (4.1)

$$\sum_{n=1}^{\infty} g_n \sin n\tau, \quad g_n = -n^{-1}\left(4 - \sum_{j=1}^{4} F_j^{\,n}\right), \quad F_j = \frac{E_j - 1}{E_j + 1}$$

is equal to $\operatorname{Im}\ln\Lambda\,(l+i\tau)$ on AC and zero on $CDFA$ if we set

$$\Lambda(z)=\exp\sum_{n=1}^{\infty} g_n \operatorname{csch} nl \operatorname{ch} nz$$

The general term of the series under the exponential has the bounds (on the real part)

$$|g_n \operatorname{ch} n\sigma \cos n\tau \operatorname{csch} nl| \leqslant |g_n| \operatorname{ch} n\sigma \operatorname{csch} nl \leqslant 4|g_n| \exp[-n(l-\sigma)]$$

since $l,\ n>0$ and $\sigma \geqslant 0$, and therefore $\operatorname{sh} nl > 1/4e^{nl}$ and $\operatorname{ch} n\sigma < e^{n\sigma}$; it is dominated by the geometric progression with ratio $e^{-(l-\sigma)}$. The imaginary part has just the same bounds. This shows the uniform convergence of the series to an analytic function in any closed region consisting of internal and boundary points of the rectangle (3.3), except for points of its right-hand vertical side.

Following Lighthill we may represent the function $\Lambda(z)$ on the contour by infinite products. On the image DF of the wall

$$\Lambda(i\tau)=\prod_{n=0}^{\infty}\left[(1-2q^{n+\frac{1}{2}}\cos\tau+q^{2n+1})^4\prod_{j=1}^{4}(1-2q^{n+\frac{1}{2}}F_j\cos\tau+q^{2n+1}F_j^2)^{-1}\right] \tag{4.2}$$

The expression $\Lambda(\sigma)$ for points of the image of the Mach circle differs only in having the quantity $\operatorname{ch}\sigma$ in place of $\cos\tau$ in (4.2). On the image of the reflected front AC we obtain the expression

$$|\Lambda(l+i\tau)|=\prod_{n=0}^{\infty}{}'\left[(1-2q^n\cos\tau+q^{2n})^4\prod_{j=1}^{4}(1-2q^nF_j\cos\tau+q^{2n}F_j^2)^{-1}\right]$$

where the prime indicates that the square root must be extracted from the first factor.

The conformal transformation of the rectangle (3.3) onto the lower half plane

$$\omega=\xi+i\eta=-\sqrt{k}\,\vartheta_2(-iz,q)\,/\,\vartheta_3(-iz,q) \tag{4.3}$$

permits the function $L(z)$ to be obtained at once. The points $A,\ C,\ D,\ F$ and $z=z_0$ are transformed into the points $\xi=\mp 1$, $\xi=\pm k$ and $\xi_0\,(z_0)$ of the real axis. The quantities ϑ_1 to ϑ_4 are theta functions [15]; the quantity k depends on the quantity q

$$k^2=1-k'^2,\ \sqrt{k'}=(1-2q+2q^4-2q^9+\ldots)\,/\,(1+2q+2q^4+2q^9+\ldots)$$

The function $L(z)$, having the necessary piecewise constant argument along the real axis, is conveniently written in the form

$$L(z)=L_0(z)L_1(z)L_2(z)=[\omega(z)-\xi_0(z_0)]\frac{-i\sqrt{k'/k}}{\sqrt{1-\omega^2(z)}}\frac{1-\omega(z)}{1+\omega(z)} \tag{4.4}$$

The product of the first two factors generalizes the corresponding functions in Lighthill's solution. In the limiting $(\gamma\to 0)$ symmetric case the Fourier coefficients contain terms generating a series converging to $-\ln L_2(z)$; this factor is introduced to simplify the expression for $\gamma\neq 0$. On the sides DF, FA and AC of the rectangle (3.3) the function (4.3) has the expression

$$\xi(i\tau)=-\sqrt{k}\frac{\vartheta_2(\tau,q)}{\vartheta_3(\tau,q)},\quad \xi(\sigma)=-\sqrt{k}\frac{\vartheta_1(\sigma,q')}{\vartheta_3(\sigma,q')},\quad \xi(l+i\tau)=\sqrt{k}\frac{\vartheta_3(\tau,q)}{\vartheta_2(\tau,q)} \tag{4.5}$$

where $\ln q\ln q'=\pi^2$. Substituting (4.5) into (4.4) gives the boundary values of the factors in the function $L(z)$. For $L_0^-(z)$ and $L_2^-(z)$ the result is obvious. For $L_1^-(z)$ on DF, FA and AC we obtain the expressions

$$L_1(i\tau) = -i\sqrt{k}\frac{\vartheta_3(\tau, q)}{\vartheta_4(\tau, q)},\ L_1(\sigma) = i\sqrt{k}\frac{\vartheta_3(\sigma, q')}{\vartheta_2(\sigma, q')},\ \ L_1(l + i\tau) = \sqrt{k}\frac{\vartheta_2(\tau, q)}{\vartheta_1(\tau, q)}$$

At an arbitrary point of the rectangle (3.3)

$$L_1(z) = -i\sqrt{k}\vartheta_3(-iz,\ q) / \vartheta_4(-iz, q)$$

5. Solution. It is not difficult, having the function $\Gamma_0(z)$, to write the solution of the original nonhomogeneous problem according to equations of a developed theory [16, 17]; a simple test confirms its correctness

$$\Gamma(z) = \Phi(z)\left[-\frac{1}{i\pi}\frac{p_4\xi_\sigma'(\sigma_G)}{\Phi(\sigma_G)}\frac{1}{\xi(\sigma_G) - \omega(z)} + cL_0(z)\right] \tag{5.1}$$

$$\Phi(z) = \Lambda(z)\, L_1(z)\, L_2(z)$$

Here the derivative ξ_σ' is given by the equation [15]

$$\xi_\sigma'(\sigma_G) = \frac{2K\, k'\vartheta_1(\sigma_G,\ q')\,\vartheta_2(\sigma_G,\ q')}{\pi k\sqrt{k}\,\vartheta_3^2(\sigma_G,\ q')} \tag{5.2}$$

$$2K/\pi = (1 + 2q + 2q^4 + 2q^9 + \ldots)^2$$

Boundary values of (5.1) are given by the expression

$$\Gamma^-(z) = \Phi^-(z)\left[-\frac{1}{i\pi}\frac{p_4\xi_\sigma'(\sigma_G)}{\Phi(\sigma_G)}\frac{1}{\xi(\sigma_G) - \xi(z)} + cL_0(z)\right] + p_4\delta(\sigma - \sigma_G) \tag{5.3}$$

It follows from (5.3) that along AC

$$\frac{\partial p}{\partial\tau} = -\operatorname{Im}\Phi(l + i\tau)\left\{c\,[\omega(l + i\tau) - \xi_0(z_0)] - \frac{c_0}{\xi(\sigma_G) - \xi(l + i\tau)}\right\} \tag{5.4}$$

where

$$\operatorname{Im}\Phi(l + i\tau) = L_1(l + i\tau)\, L_2(l + i\tau)\,|\Lambda(l + i\tau)|\, b(\tau) / \sqrt{b^2(\tau) + 1}$$

$$c_0 = \frac{p_4\xi_\sigma'(\sigma_G)\,\vartheta_2(\sigma_G, q')}{\pi\sqrt{k}\,\Lambda(\sigma_G)\, L_2(\sigma_G)\,\vartheta_3(\sigma_G, q')}$$

All the functions entering here are real, and are determined in Sects. 3 and 4. Substitution of (5.4) and the quantity $y = m\operatorname{tg}\theta$, with regard to (3.4) and (3.6) and with the introduction of the notation

$$c_1 = \frac{\sqrt{1 - m^2}\, M_1(\delta_2 - \delta_1)}{B},\quad \Psi(\tau) = \frac{|\Lambda(l + i\tau)|\, L_2(l + i\tau)\, b(\tau)}{\sqrt{b^2(\tau) + 1}\,\vartheta_1(\tau, q)},\ c_2 = p_5 - p_4$$

leads to a system having the solution

$$c = \frac{c_1 I_4 - c_2 I_2 - c_0(I_6 I_4 - I_5 I_2)}{I_1 I_4 - I_2 I_3},\quad \xi_0(z_0) = -\frac{1}{\sqrt{k}}\frac{c_1 I_3 - c_2 I_1 - c_0(I_6 I_3 - I_5 I_1)}{c_1 I_4 - c_2 I_2 - c_0(I_6 I_4 - I_5 I_2)} \tag{5.5}$$

The integrals

$$I_1 = \sqrt{1 - m^2}\int_0^\pi \vartheta_3(\tau, q)\,\Psi(\tau)\frac{d\tau}{y(\tau)},\quad I_2 = \sqrt{1 - m^2}\int_0^\pi \vartheta_2(\tau, q)\,\Psi(\tau)\frac{d\tau}{y(\tau)},$$

$$I_3 = \int_0^\pi \vartheta_3(\tau, q)\,\Psi(\tau)\, d\tau,\quad I_4 = \int_0^\pi \vartheta_2(\tau, q)\,\Psi(\tau)\, d\tau$$

$$I_5 = \int_0^\pi \frac{\vartheta_2(\tau, q)\,\Psi(\tau)\,d\tau}{\xi(\sigma_G) - \xi(i\tau)}, \quad I_6 = \sqrt{1-m^2}\int_0^\pi \frac{\vartheta_2(\tau, q)\,\Psi(\tau)}{\xi(\sigma_G) - \xi(i\tau)}\,\frac{d\tau}{y(\tau)}$$

are determined numerically. Knowing $\xi_0(z_0)$, it is possible to determine z_0 by means of Eqs. (4.5) and a table of theta functions, but this is not required for the solution; it is only necessary to substitute into (5.1) and (5.3) the values of c and $\xi_0(z_0)$ obtained from (5.5).

Note 5.1. The a priori form of solution given in the work of the author [2] does not reflect the presence of the third zero of function Γ_0 on the boundary of the rectangular region. A solution of the homogeneous problem that contains it is obtained by multiplying the function Γ_0 appearing there by the factor $(\omega - \xi_0)/(\omega - 1/k)^2$ and then determining ξ_0 simultaneously with the constant multiplier under the same normalizing conditions and in the same way as in the present paper.

6. Pressure on wall. The pressure distribution is determined by integrating the partial derivative $\partial p/\partial\tau$ along the image of the wall DF in the z-plane

$$\frac{\partial p}{\partial \tau} = -\operatorname{Im}\Gamma(i\tau) = \Lambda(i\tau)\,L_2(i\tau)\,\frac{\vartheta_3(\tau, q)}{\vartheta_4(\tau, q)}\left[cL_0(i\tau) - \frac{\sqrt{k}\,c_0}{\xi(\sigma_G) - \xi(i\tau)}\right]$$

and calculating the coordinate $r = |(M\cos\tau - 1)/(M - \cos\tau)|$, measured from point E along the wall in the direction of point F (for $\tau < \arccos M^{-1}$) or in the direction of point D.

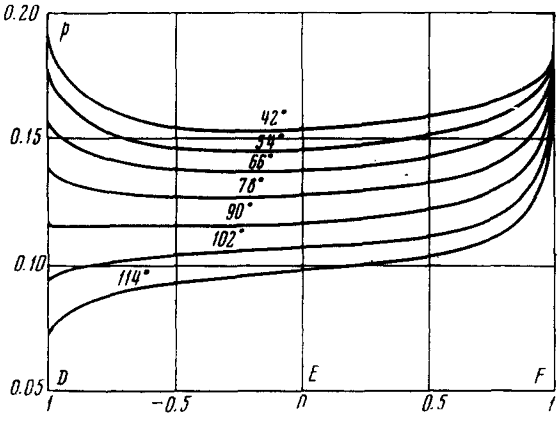

Fig. 2

Figure 2 shows the results of calculations carried out for $\varepsilon = 0.1465$, $\beta = 15.5°$ and $M_\infty = 2.207$. The values of the angle χ are shown on the graph.

It is seen how significant changes in the magnitude, and thus a displacement of the extremal value of the quantity p to point D and beyond it with increasing angle χ, is associated with the position of the point z_0.

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